

The Multiple Point Principle and Higgs Bosons

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Abstract

The multiple point principle (MPP), according to which several vacuum states with the same energy density exist, is put forward as a fine-tuning mechanism predicting the ratio between the fundamental and weak scales in the Standard Model (SM) and in its two Higgs doublet extension (2HDM). Using renormalization group equations for the SM, we obtain the effective potential in the 2-loop approximation and investigate the existence of its postulated second minimum at the fundamental scale. In the SM an exponentially huge ratio between the fundamental (Planck) and electroweak scales results: $\frac{\Lambda_{fund}}{\Lambda_{ew}} \sim e^{40}$. But in the 2HDM the fundamental scale Λ can vary from 10 TeV up to the Planck scale. Using the MPP, we predict the masses of the Higgs bosons.

1. Cosmological Constant and the Multiple Point Principle

In the present talk we suggest two scenarios:

I) The first scenario [1-4] considers only the pure Standard Model (SM) with one Higgs boson. In this scenario we obtain an exponentially huge ratio between the fundamental (Planck) and electroweak scales: $\frac{\Lambda_{fund}}{\Lambda_{ew}} \sim 10^{17} \sim e^{40}$.

II) The second scenario [5] concerns to the general two Higgs doublet extension of the SM, in which the fundamental scale Λ can vary from 10 TeV up to the Planck scale: $\Lambda \sim 10^{19}$ GeV .

In such scenarios it is reasonable to assume the existence of a simple postulate which helps us to explain the SM parameters: couplings, masses and mixing angles. In our

model such a postulate is based on a phenomenologically required result in cosmology: the cosmological constant is zero, or approximately zero, meaning that the vacuum energy density is very small. A priori it is quite possible for a quantum field theory to have several minima of the effective potential as a function of its scalar fields. Postulating zero cosmological constant, we are confronted with a question: is the energy density, or cosmological constant, equal to zero (or approximately zero) for all possible vacua or it is zero only for that vacuum in which we live? This assumption would not be more complicated if we postulate that all the vacua which might exist in Nature, as minima of the effective potential, should have approximately zero cosmological constant. This postulate corresponds to the Multiple Point Principle (MPP) developed in [6] (see also [7]): **there are many vacua with the same energy density or cosmological constant, and all cosmological constants are zero, or approximately zero.**

Using the MPP, we predict the masses of the Higgs bosons.

2. The Higgs boson mass value in the SM

The success of the SM strongly supports the concept of the spontaneous breaking of symmetry $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, which is achieved by the Higgs mechanism, giving masses of the gauge bosons W^\pm , Z , the Higgs boson and fermions. This mechanism, in its minimal version, requires the introduction of a single doublet of scalar complex Higgs fields and leads to the existence of a neutral massive particle - the Higgs boson.

Recently the experimental lower limit on the Higgs mass of 115.3 GeV was set by the unsuccessful search at LEP II. The energy interval from 100 GeV to 200 GeV will be thoroughly examined at the upgraded Tevatron, LHC and LC. These machines have a good chance to discover the Higgs boson in the near future.

With one Higgs doublet of $SU(2)_L$, we have the following tree-level Higgs potential:

$$V^{(0)} = -m^2 \Phi^\dagger \Phi + \frac{\lambda}{2} (\Phi^\dagger \Phi)^2. \quad (1)$$

The vacuum expectation value of the Higgs field Φ is:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (2)$$

where $v = \sqrt{\frac{2m^2}{\lambda}} \approx 246$ GeV. The masses of gauge bosons W and Z , fermions with flavor f and the physical Higgs boson are given by the VEV parameter v :

$$M_W^2 = \frac{1}{4} g^2 v^2, \quad M_Z^2 = \frac{1}{4} (g^2 + g'^2) v^2, \quad m_f = \frac{1}{\sqrt{2}} h_f v, \quad M_H^2 = \lambda v^2, \quad (3)$$

where h_f are the Yukawa couplings with flavor f .

In our paper [3] we have calculated the 2-loop effective potential in the limit $\phi^2 \gg v^2$, $\phi^2 \gg m^2$, using the SM renormalization group equations in the 2-loop approximation [8]. Assuming the existence of the two minima of the effective potential in the simple SM, we have taken the cosmological constants for both vacua equal to zero, in accord with the Multiple Point Principle.

3. The Multiple Point Principle requirements

The MPP requirements for the two degenerate minima in the SM are given by the following equations:

$$V_{eff}(\phi_{min1}) = V_{eff}(\phi_{min2}) = 0, \quad V'_{eff}(\phi_{min1}) = V'_{eff}(\phi_{min2}) = 0, \quad (4)$$

$$V''_{eff}(\phi_{min1}) > 0, \quad V''_{eff}(\phi_{min2}) > 0, \quad (5)$$

where

$$V'(\phi) = \frac{\partial V}{\partial \phi^2}, \quad V''(\phi) = \frac{\partial^2 V}{\partial (\phi^2)^2}. \quad (6)$$

As was shown in [1] the degeneracy conditions of MPP give the following requirements for the existence of the second minimum in the limit of high $\phi^2 \gg m^2$:

$$\lambda(\phi_{min2}) = 0, \quad \text{and} \quad \lambda'(\phi_{min2}) = 0, \quad (7)$$

what means $\beta_\lambda(\phi_{min2}, \lambda = 0) = 0$.

Using these requirements and the renormalization group flow, C.D.Froggatt and H.B.Nielsen [1] computed quite precisely the top quark (pole) and Higgs boson masses (see also the detailed calculations in [3]):

$$M_t = 173 \pm 4 \text{ GeV} \quad \text{and} \quad M_H = 135 \pm 9 \text{ GeV}. \quad (8)$$

4. Two Higgs doublet extension of the SM

There are no strong arguments for the existence of just a single Higgs doublet, apart from simplicity. In the present talk we consider the implementation of the MPP in the general 2HDM, without any symmetries imposed beyond those of the SM gauge group. The most general renormalizable $SU(2) \times U(1)$ gauge invariant potential of the 2HDM is given by

$$\begin{aligned} V_{eff}(H_1, H_2) = & m_1^2(\Phi)H_1^+H_1 + m_2^2(\Phi)H_2^+H_2 - [m_3^2(\Phi)H_1^+H_2 + h.c.] \\ & + \frac{\lambda_1(\Phi)}{2}(H_1^+H_1)^2 + \frac{\lambda_2(\Phi)}{2}(H_2^+H_2)^2 + \lambda_3(\Phi)(H_1^+H_1)(H_2^+H_2)^2 \end{aligned}$$

$$\begin{aligned}
& +\lambda_4(\Phi)|H_1^+H_2|^2 + [\frac{\lambda_5(\Phi)}{2}(H_1^+H_2)^2 + \lambda_6(\Phi)(H_1^+H_1)(H_1^+H_2) \\
& +\lambda_7(\Phi)(H_2^+H_2)(H_1^+H_2) + h.c.],
\end{aligned} \tag{9}$$

where

$$H_i = \begin{pmatrix} \chi_i^+ \\ (H_i^0 + iA_i^0)/\sqrt{2} \end{pmatrix}.$$

The number of parameters in the 2HDM compared with the SM grows from 2 to 10.

We suppose that mass parameters m_i^2 and Higgs self-couplings λ_i of the effective potential only depend on the overall sum of the squared norms of the Higgs doublets, i.e.

$$\Phi^2 = \Phi_1^2 + \Phi_2^2, \quad \Phi_i^2 = H_i^+H_i = (H_i^0)^2 + (A_i^0)^2 + |\chi_i^+|^2. \tag{10}$$

The running of these couplings is described by the 2HDM renormalization group equations [8], where the renormalization scale is replaced by Φ .

At the physical minimum of the 2HDM scalar potential the Higgs fields develop vacuum expectation values (VEVs)

$$\langle \Phi_1 \rangle = \frac{v_1}{\sqrt{2}}, \quad \langle \Phi_2 \rangle = \frac{v_2}{\sqrt{2}}, \tag{11}$$

breaking $SU(2) \times U(1)$ gauge symmetry and generating masses of all bosons and fermions.

Here the overall Higgs norm: $\langle \Phi \rangle = \sqrt{v_1^2 + v_2^2} = v = 246 \text{ GeV}$ is fixed by the electroweak scale. At the same time the ratio of the Higgs VEVs remains arbitrary, and it is convenient to introduce $\tan \beta = v_2/v_1$.

Assuming no CP, nor charge violation at the vacuum, the 2HDM involves five physical states of the Higgs bosons:

- 1) The charged Higgs bosons χ^\pm have masses m_{H^\pm} .
- 2) One CP-odd Higgs boson A_2^0 has a mass m_A .
- 3) The two CP-even scalars have masses m_H and m_h .

The last one is a mass of the lightest Higgs particle.

5. Implementation of the MPP in the 2HDM

In this talk we present 2HDM supplemented by the MPP assumption. We require that at some high energy scale $M_Z \ll \Lambda \lesssim M_{Pl}$, which we shall refer as the MPP scale Λ , the largest set of degenerate vacua is realized in 2HDM. In compliance with the MPP, these vacua and the physical one must have the same energy density.

We expected that the 2HDM effective potential, depending on the norms $|H_1|$ and $|H_2|$, had to have two rings of minima in the Mexican hat with the same vacuum energy

density. The radius of the little ring is at the electroweak scale v , while the radius of the big one is Λ . In this case, imposing the MPP conditions, we had to have that all the self-couplings should vanish at the MPP scale:

$$\lambda_1(\Lambda) = \lambda_2(\Lambda) = \lambda_3(\Lambda) = \lambda_4(\Lambda) = \lambda_5(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0. \quad (12)$$

But this case is in contradiction with the 2HDM renormalization group equations for couplings. The position of vacua depends on $\tan \beta$. In addition, the vacuum stability conditions must be satisfied.

For example, there exists the MPP solution for the 2HDM vacua at the scale Λ which results in a set of degenerate vacua:

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ \Phi_1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} \Phi_2 \\ 0 \end{pmatrix}, \quad (13)$$

where $\Phi_1^2 + \Phi_2^2 = \Lambda^2$, and

$$\lambda_1(\Lambda) = \lambda_2(\Lambda) = \lambda_3(\Lambda) = 0, \quad \lambda_5(\Lambda) = \lambda_6(\Lambda) = \lambda_7(\Lambda) = 0, \quad (14)$$

$$\beta_{\lambda_1}(\Lambda) = \beta_{\lambda_2}(\Lambda) = \beta_{\lambda_3}(\Lambda) = 0 \quad (15)$$

with the vacuum stability condition $\lambda_4(\Lambda) > 0$.

This MPP solution allows to evaluate the Yukawa couplings of the third generation at the electroweak scale in terms of only one parameter $h_t(\Lambda)$. The values of $\tan \beta$ and $h_t(\Lambda)$ can be fitted so that the correct values of the top-quark and τ -lepton masses are reproduced:

$$m_t(M_t) = \frac{h_t(M_t)v}{\sqrt{2}} \sin \beta, \quad m_\tau(M_t) = \frac{h_\tau(M_t)v}{\sqrt{2}} \cos \beta.$$

Using experimentally given values:

$$\alpha_3 \approx 0.117, \quad m_t(M_t) \approx 165 \text{ GeV}, \quad m_\tau(M_t) \approx 1.78 \text{ GeV},$$

we obtain that $\tan \beta$ is confined around 50.

Finally the Higgs masses depend only on the two parameters: Λ and m_A . We have made a detailed numerical analysis of the MPP constraints on the Higgs spectrum in the 2HDM for high energy scales ranging from $\Lambda = 10 \text{ TeV}$ to $\Lambda = \mathbf{M}_{\mathbf{Pl}}$.

We present the following examples:

1) For $\Lambda = 10 \text{ TeV}$ and $m_A = 400 \text{ GeV}$, we have:

$$\tan \beta \approx 49.8, \quad m_h \approx 69 \text{ GeV}, \quad m_{H^\pm} \approx 360 \text{ GeV}, \quad m_H \approx 399 \text{ GeV}.$$

2) For $\Lambda = 10^8$ GeV and $m_A = 400$ GeV : $\tan\beta \approx 50$, $m_h \approx 115$ GeV, what corresponds to the LEP II experimental lower limit for the light Higgs boson, and $m_{H^\pm} \approx 387$ GeV, $m_H \approx 399.5$ GeV.

3) For $\Lambda = M_{Pl} = 1.22 \cdot 10^{19}$ GeV and $m_A = 400$ GeV : $\tan\beta \approx 50.3$, $m_h \approx 137 \pm 10$ GeV, what is quite close to the MPP prediction of the Higgs boson mass in the SM [1] (see Eq.(8)). Here we also have: $m_{H^\pm} \approx 404$ GeV, $m_H \approx 400$ GeV.

We see that the masses of Higgs bosons grow with increasing Λ .

In conclusion I want to emphasize that we tried to construct in [5] a new simple MPP inspired non-supersymmetric two Higgs doublet extension of the SM.

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